We have benthic cover (coral, maroalgae, CCA) expressed as an average (and standard deviation) for each of a series of discrete 100m wide grid cells around each island, and we have two predictors (surface wave, internal wave energy) specific to each of these cells (although the predictors are generated at 1km scale for internal waves and 600m – 1km for surface waves, so end up with adjacent grid cells with very similar values). So, if we denote island- and sector-specific benthic cover (macroalgal cover for example) as , where i is island and j is sector, we can assume (as a first pass) that

i.e. the observed macroalgal cover can be modelled as deriving from a normal distribution (realistically, it might need to be logistically transformed to meet this requirement, but that is details). This effectively just models the observation/residual side of the model, with the expected grid cell-specific macroalgal cover.

Denoting internal wave energy as and surface wave energy as we can write the model for the expected macroalgal cover as

where f() is a link function, is the island specific intercept (this could be modelled as a random effect i.e. ), is the regression coefficient modelling the effect of internal wave energy, specific to each island, is the regression coefficient modelling the effect of surface wave energy, specific to each island, and is a spatially correlated random effect for each grid cell. The effects of internal waves and surface waves are hierarchical in that each island has its own coefficient, but they derive from a parent population, i.e.

This allows each island to “respond” differently, but according to some global law. One of the ways to then examine the effect of population status on these effects is to see whether the island specific values are lower/higher on populated islands than unpopulated islands, or whether values cluster within regions (or even are super far from the “mean” for some islands). As a first pass, we can specify a single ‘set’ (i.e. all islands have the same central tendency regarding these regression coefficients), but if there is evidence that islands with more people are very different, then you could specify two populations, one for populated islands and one for remote islands, with each set having a different central tendency (could get more complicated if use the continuous number of people). These two models could then be compared using BIC, DIC etc.

There’s also the idea that adjacent islands might “respond” similarly, such that are correlated according to inter-island distance. For a first pass we can ignore this, and if it becomes an issue we could model it using a Gaussian process (see later explanation), but this would make the model more complicated.

The spatially correlated random effects account for the fact that grid cells around islands are contiguous, and so they are likely autocorrelated according to separation. This can be accounted for by assuming that the random effects come from a multivariate normal distribution:

where is a covariance matrix, whose elements are defined by

where is an identity matrix, is the distance between cells k and l, control the magnitude of variation, and controls the rate at which covariance decays. This is such that the diagonal elements have variance (i.e. the random effects variance) , whereas the off-diagonal elements (i.e. cross correlated) have variance , such that covariance decays with separation. The one advantage of this is that you are accounting for the covariance among cells, but also you can have the same parameters for all islands (the random effects are then just alternate draws from effectively the same distribution). This is known as a gaussian process, and has been used fairly extensively in spatial statistics so shouldn’t upset too many people.

However, none of this addresses the fact that each cell is an average across varying numbers of observations, so each sector average has a different variance, as well as weight of data backing it up. One potential way to account for this is to reduce the response variables down to the ‘raw data’, i.e.

and assume that the raw observations are modelled by spatially correlated random effects according to an MVN distribution.